AXIOMATIC FOUNDATIONS OF THE COMPLEX NUMBER SYSTEM.

As complex number defined as an ordered pair (a,b) of real number a, and b object to certain operation-al definition, which turn out to be those above. These definitions are as follows, where all letters represent real numbers.

A.) Equality $(a,b) = (c,d)$ iff $a = c$
$b = d$.

B.) Sum $(a,b) + (c,d) = (a+c, b+d)$

C.) Product $(a,b)\cdot(c,d) = (ac-bd, ad+bc)$

$m(a,b) = ma, mb$.

Theorem I Suppose z_1, z_2, z_3 belong to the set S of C.N. Then

1.) z_1+z_2 and $z_1 z_2$ belong to S Closure law

2. $z_1 + z_2 = z_2 + z_1$ Commutative law of Addition

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3) $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$
Associative law of addition

4) $z_1 z_2 = z_2 z_1$ Commutative law of multiplication.

5) $z_1(z_2 z_3) = (z_1 z_2) z_3$ Associative law of multiplication

6) $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$
Distributive law

7) $z_1 + 0 = 0 + z_1$, & $z_1 \cdot 1 = 1 \cdot z_1 = z_1$
0 is called the identity w.r.t addition & 1 is called the identity w.r.t multiplication.

8) For any ~~number~~ complex number z_1 there is a unique number z in S such that $z + z_1 = 0$ where z is called inverse of z_1 w.r.t addition and is denoted by $-z_1$}

9. For any $z_1 \neq 0$ there is a unique number z in S such that $z_1 z = z z_1 = 1$

POLAR FORM OF C.N

If P is a point in the complex plane corresponding to the complex number (x, y) or $x+iy$, then see fig I

$x = r\cos\theta$, $y = r\sin\theta$ where $r = \sqrt{x^2+y^2} = |x+iy|$ is called the modulus or absolute value of $z = x+iy$ [denoted by mod z or $|z|$ and θ called the amplitude or argument of $z = x+iy$ is the angle line OP makes with the positive x-Axis.

It follows that

$$z = x+iy = r(\cos\theta + i\sin\theta)$$

which is called the polar form of the complex number and r and θ are called polar coordinates.